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**SEMESTER TWO**

**MATHEMATICS**

**SPECIALIST**

**UNITS 1 & 2**

**2017**

**SOLUTIONS**

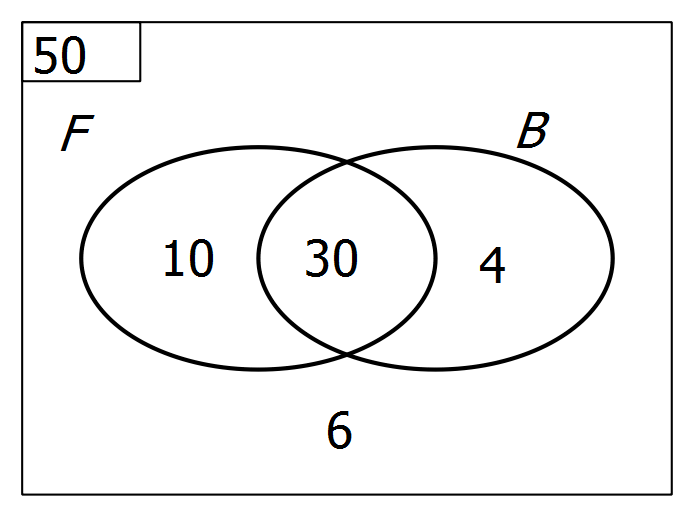
***Calculator−free Solutions***

1. (a) 1000 = 310 + 650 + 440 – 170 – 150 – 180 + *x* ✓

*x* = 100 ✓

(b) 150 – 100 = 50 ✓

(c) (i) 44 ✓

 (ii)

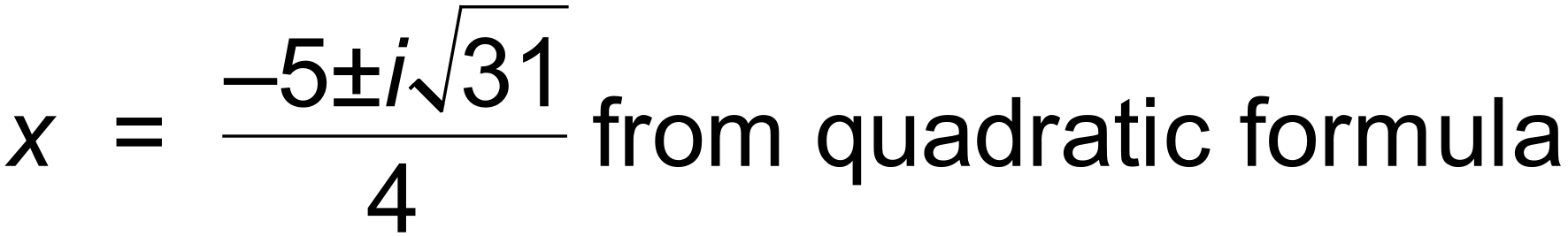
∴ n(B) = 34 ✓✓ [6]

2. (a) (i) Substitute *z* = 2i to get (2i)4 – 2(2i)3 + 7(2i)2 –8(2i) + 12

which reduces to 0 ✓

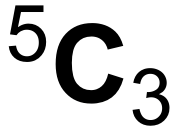
(ii) *z* = –2i (the conjugate) is the other root. ✓

(b)  ✓

∴  ✓✓ [5]

3. (a) (i)  ✓✓

(ii)  This statement is false ✓✓

(b) (i)  = 10 ✓✓

(ii) 2 x 4! = 48 ✓✓ [8]

4. (a) *p* = 4, *q* = 0.2 ✓✓

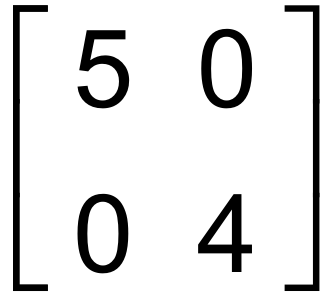
(b) *y* = 1 – *x* becomes y = 4[ 1 – 0.2 *x*]

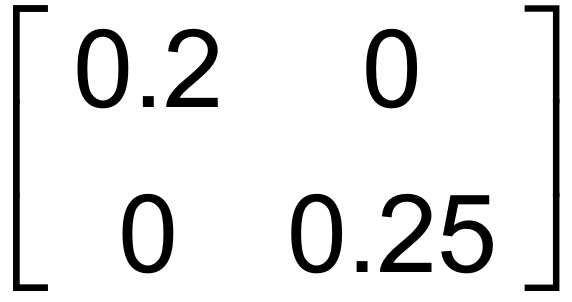
i.e. *y* = 4 – 0.8*x* ✓

or, if (c) is done before (b), gradient is – 0.8 and intercept is 4

∴ *y* = 4 – 0.8*x*

(c) A′ = ( 5, 0 ) and B′ = ( 0, 4 ) ✓✓

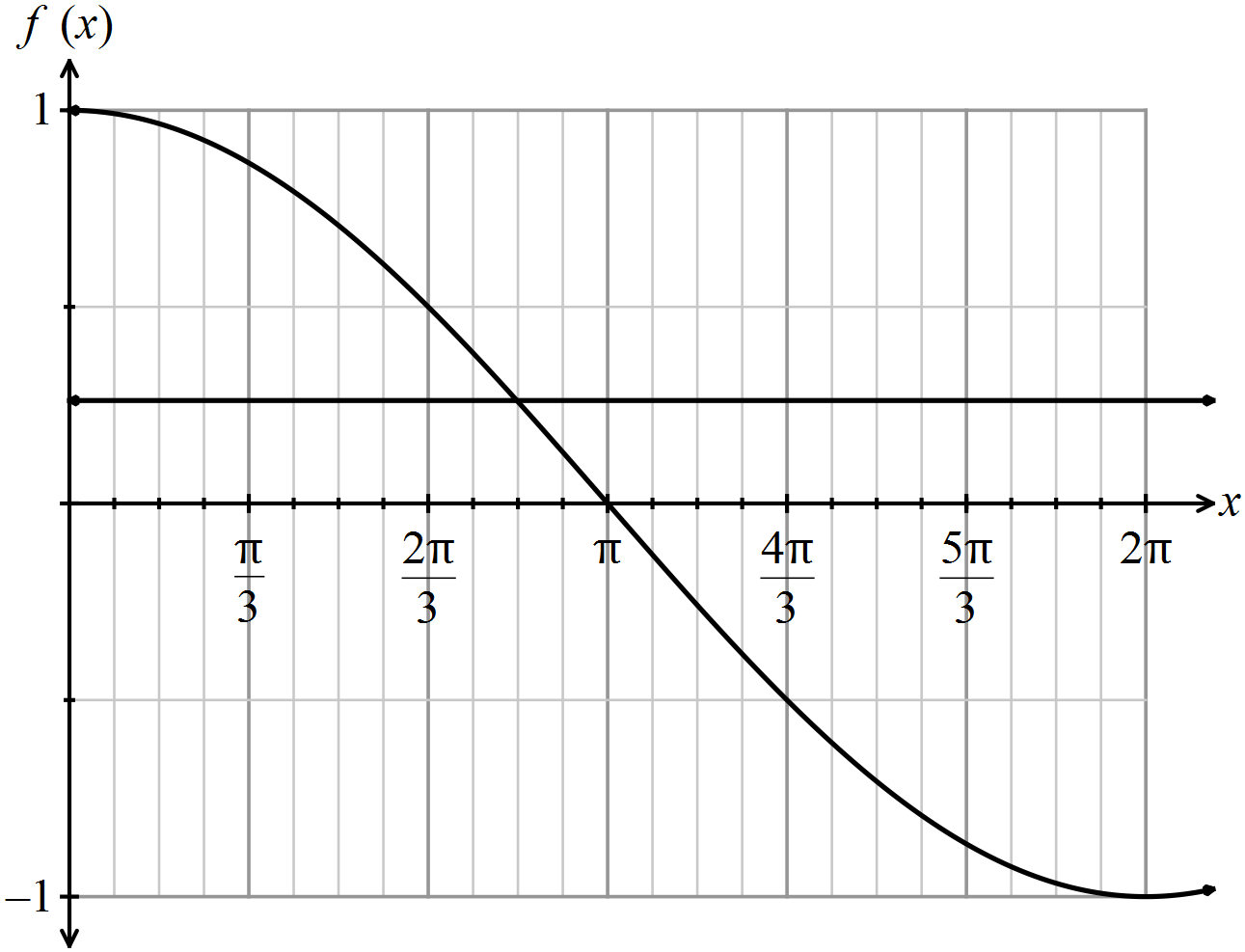
(d)  ✓✓

(e)  ✓✓

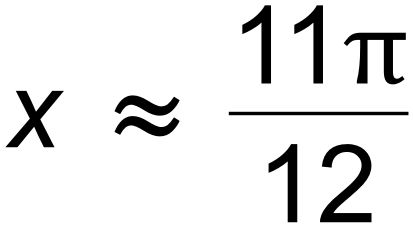
(f) Reflection across *y* axis

i.e. *g*(*x*) becomes – *g*(*x*) ✓✓

(g) A = 0.5 ( *ms* – *rn* ) ✓ [12]

5. (a)

✓✓

(b) line✓

of intersection and accuracy ✓✓

(c) sin *x* ✓

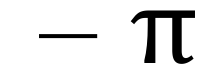
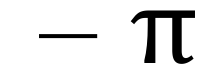
(d) 2cos *x* . sin *x* = sin 2*x* ✓✓ [8]

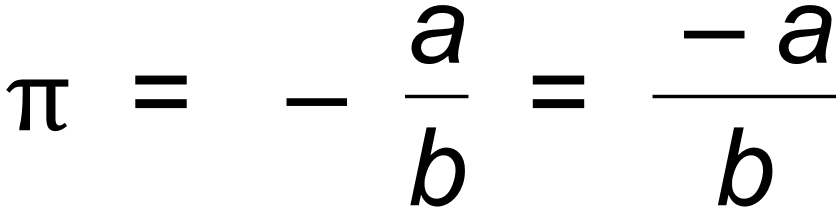
6. (a) Let the numbers be 2*k* – 1, 2*k* + 1, 2*k* + 3, 2*k* + 5, 2*k* + 7 ✓

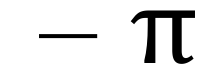
2*k* − 1 + 2*k* + 1 + 2*k* + 3 + 2*k* + 5 + 2*k* + 7

= 10*k* + 15 ✓

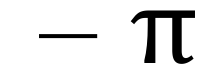
Since 10*k* + 15 = 5(2*k* + 3) then divisible by 5. ✓

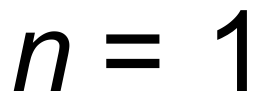
(b) Assume that  is rational, hence  =  ✓

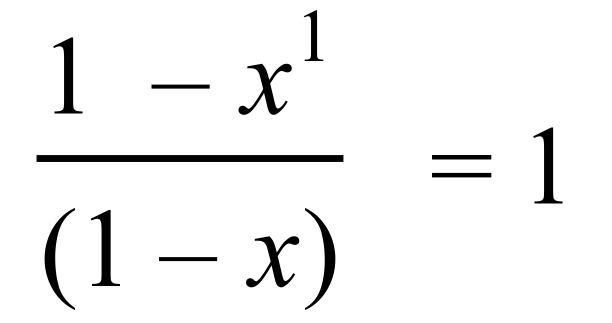
∴  ✓

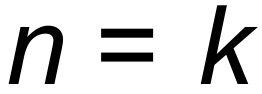
But – *a* and *b* are integers, so  is rational. ✓

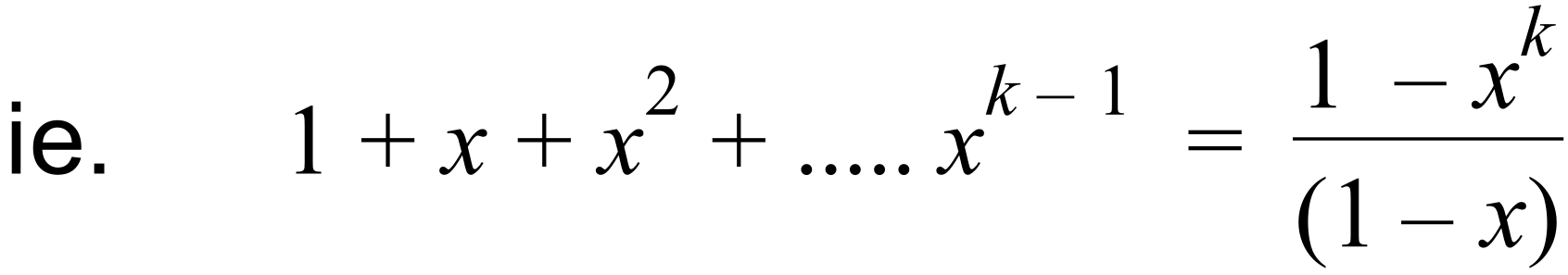
This contradicts the supposition, and

therefore by contradiction  must be irrational. ✓ [7]

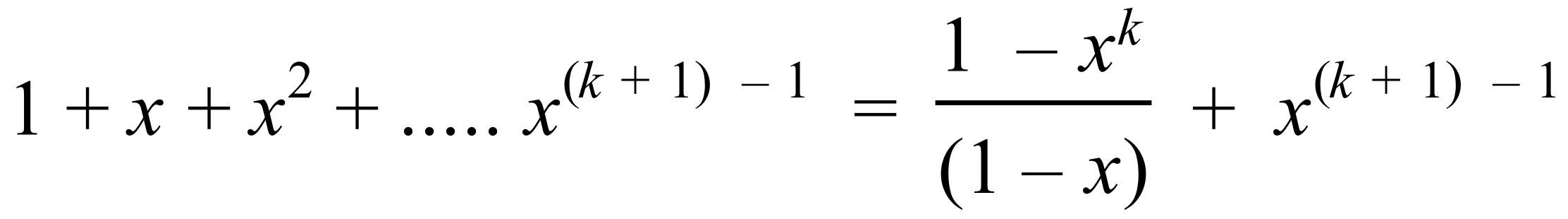
7. For :

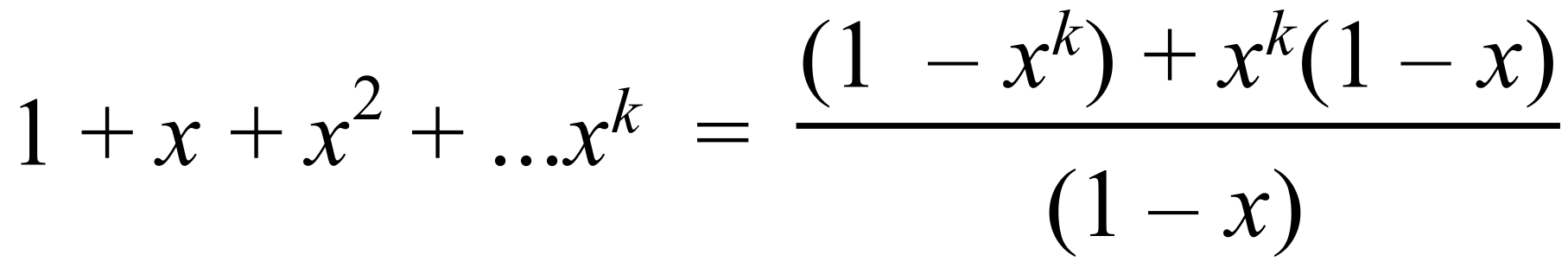
 ∴ True for n = 1 ✓

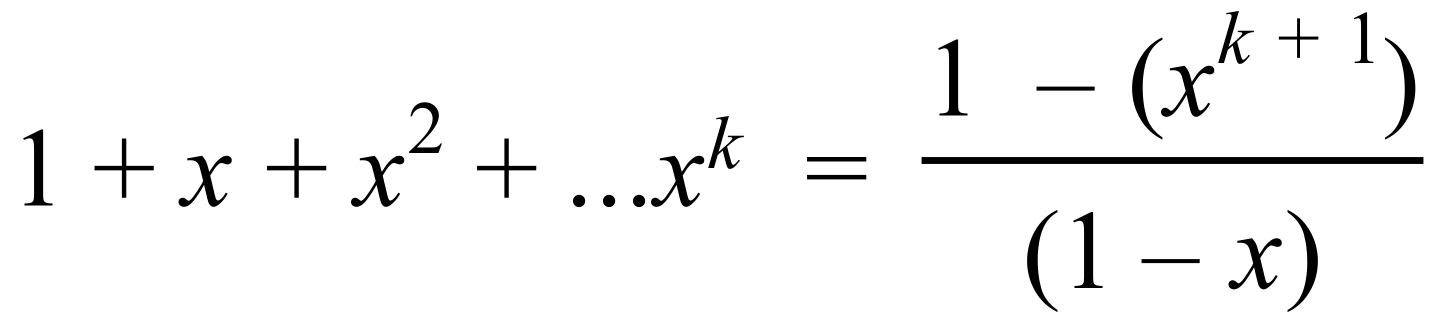
Assume true for :

ie  ✓

Prove true for :

Proof: 

 ✓

 as required ✓

Therefore, True for *n* = *k* + 1, and since true for *n* =1,

true for all whole numbers. ✓ [5]

***Calculator−assumed Solutions***

8. *wz* = (2 + *a*i)(3*b* + i) = 4

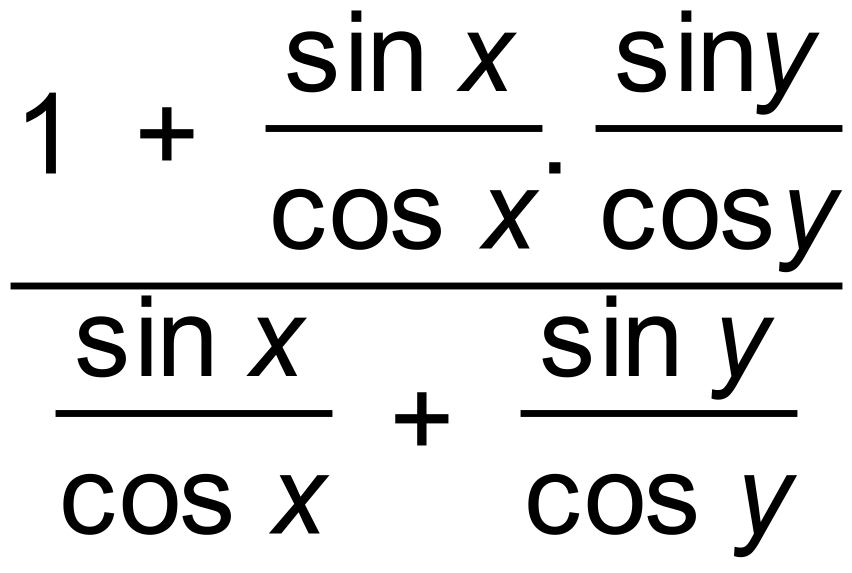
∴ 6*b* + 2i + 3*ab*i – *a* = 4 ✓

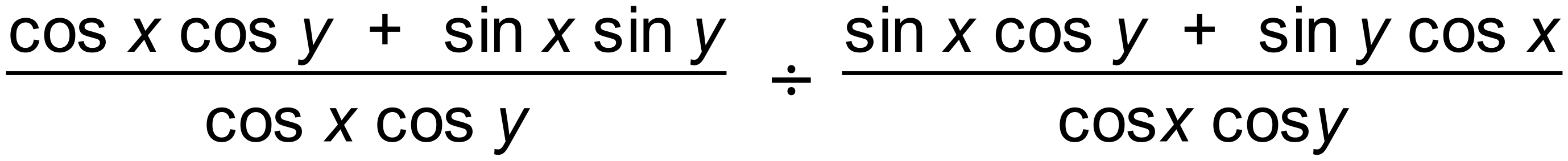
∴ 6*b* – *a* = 4 and 2 + 3*ab* = 0 ✓

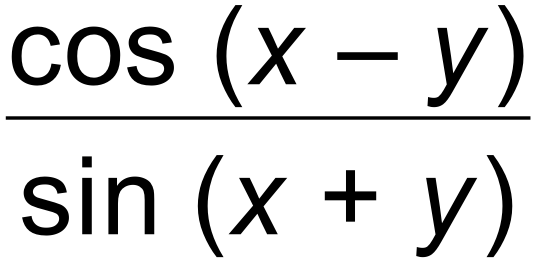
∴ 2 +3(6*b* – 4)(*b*) = 0 ✓

∴ 9*b*2 – 6*b* +1 = 0 ✓

∴ *b* =  and *a* = –2 ✓ [5]

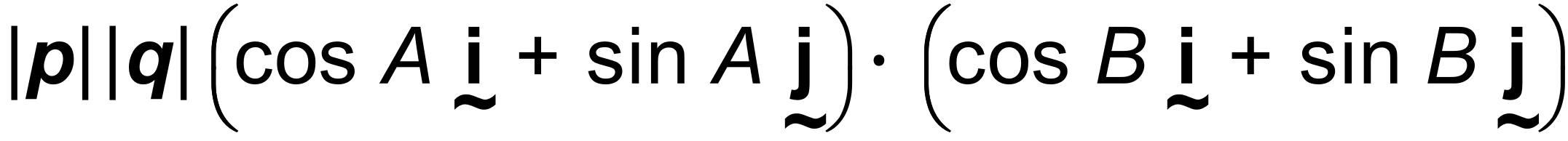
9. (a) RHS =  ✓

=  ✓✓

=  = LHS ✓

(b) (i)  ✓

and  ✓

=  ✓

= |***p***| |***q***| [ cos *A* cos *B* + sin *A* sin *B* ] ✓

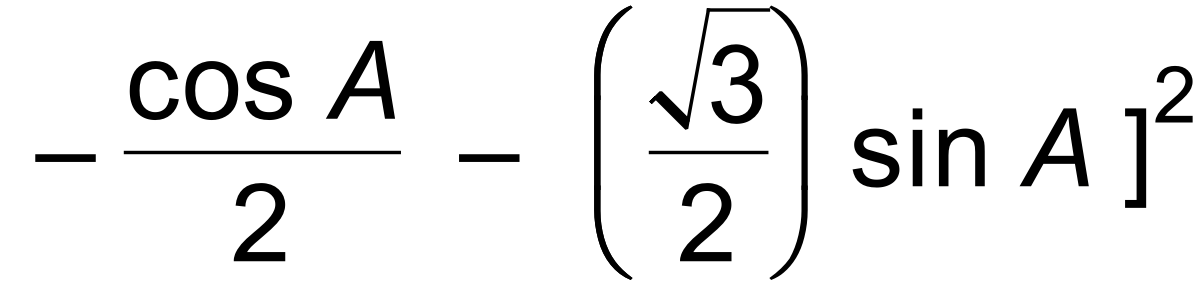
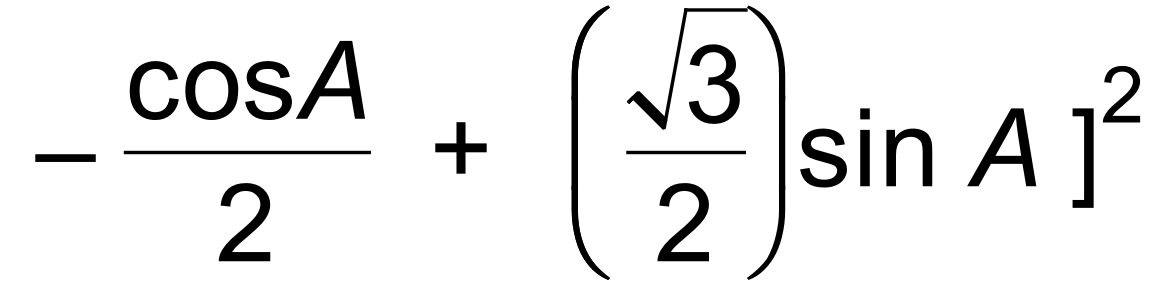


(ii) cos(*A* + *B*) = cos(*A* – (–*B*)) = cos *A* cos (–*B*) + sin *A* sin (–*B*)✓

= cos *A* cos *B* – sin *A* sin *B* ✓

(iii) cos2 *A* + [cos(120 + *A*)]2 +[cos(120– *A*)]2

= cos2 *A* + [cos 120cos *A* – sin120sin *A*]2 +[cos120cos *A* + sin120sin *A*]2✓

= cos2 *A* + [  + [  ✓

= 1.5 cos2 *A* + 1.5 sin2 *A* ✓

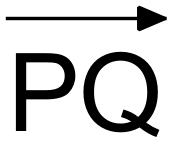
= 1.5 [13]

10. (a) (3**a** – **b**)  (**a** + 3**b**) = 3**a**  **a** – **b**  **a** + 9**a**  **b** – 3**b**  **b** ✓

= 3|**a**|2 + 8**a****b** – 3|**b**|2 ✓

= 3 + 8|**a**||**b**|cos  – 3 ✓

= 8cos as required ✓

(b)  = (4,1) ✓

Unit vector on *x* axis = (1,0) ✓

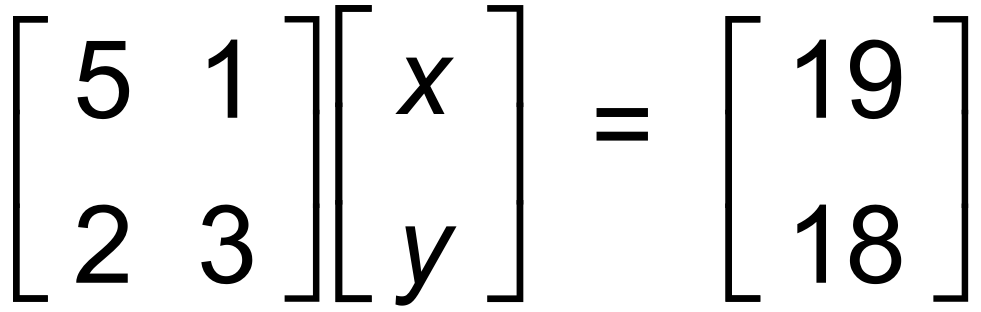
Length of projection = | PQ.  | = |4| = 4 ✓ [7]

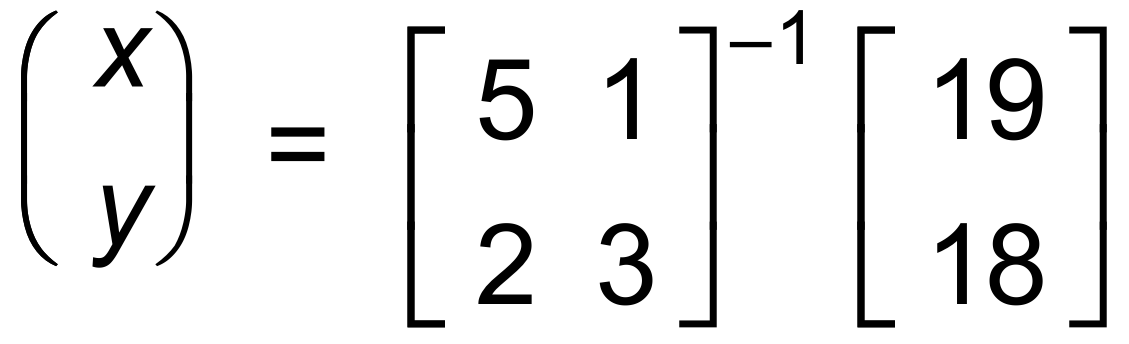
11. (a) Let the cost of a bottle of orange concentrate cost *x*

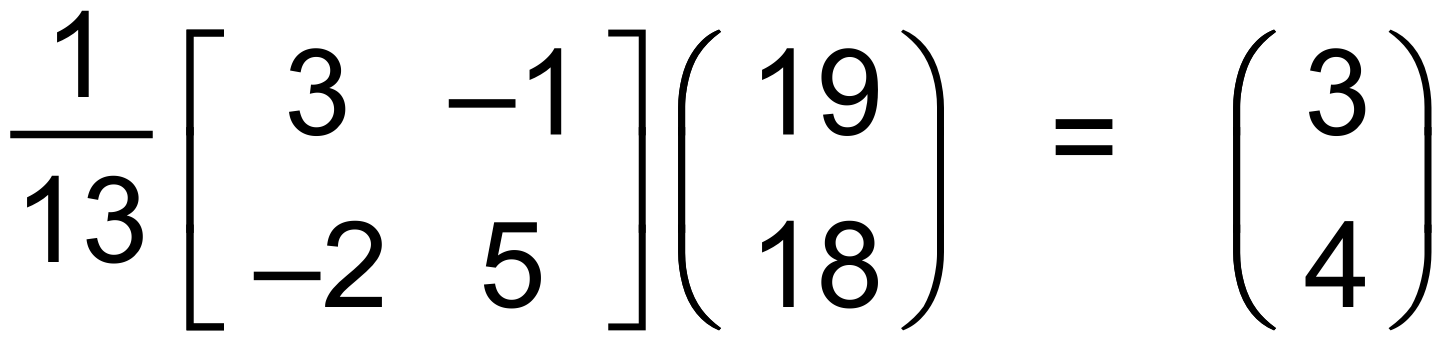
Let the cost of a bottle of banana concentrate cost *y*

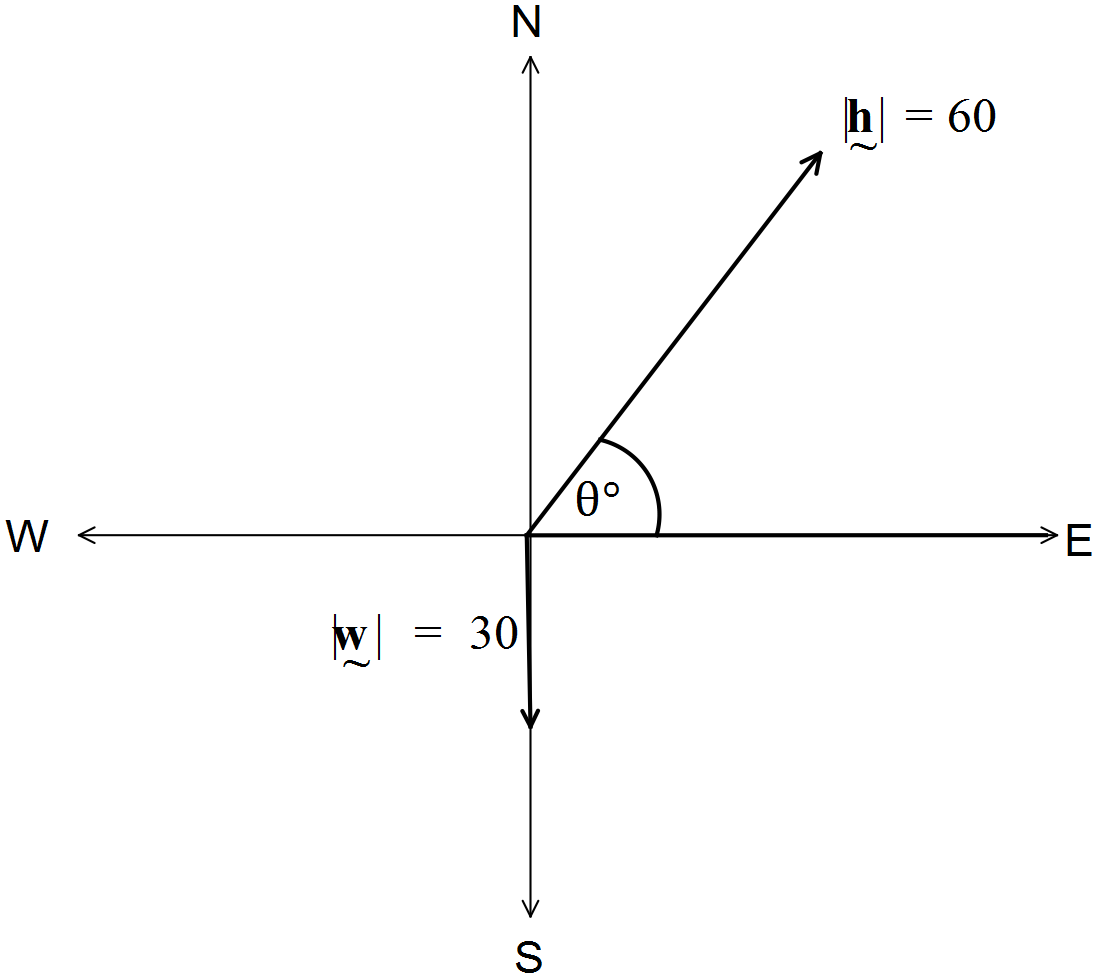
5*x* + 1*y* = 19

2*x* + 3*y* = 18 ✓

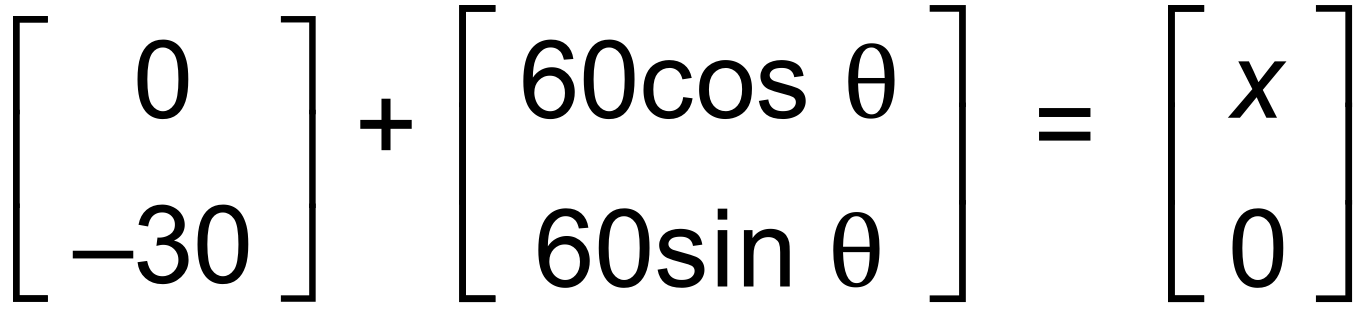
(b)  ✓

∴  ✓

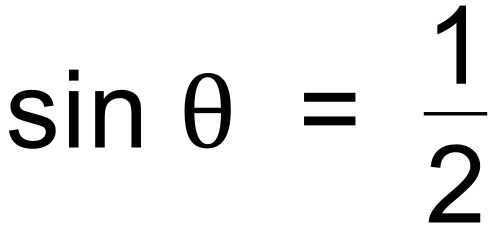
=  ✓ [4]

12. (a)

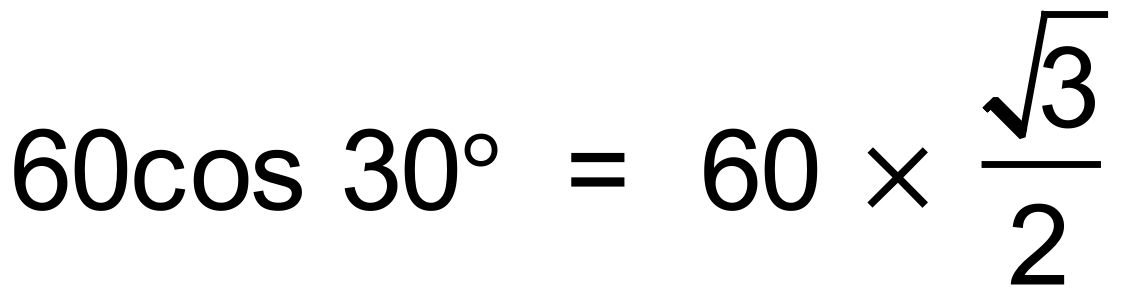
✓✓

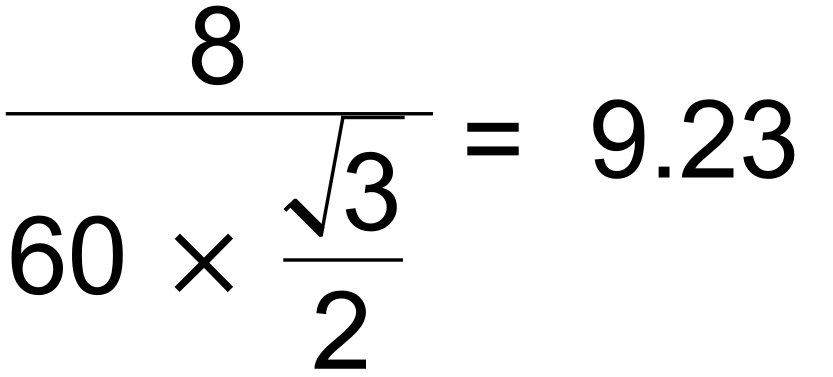
(b)  ✓✓

∴  ✓

∴ 

∴ = 30   Bearing is 060 T ✓

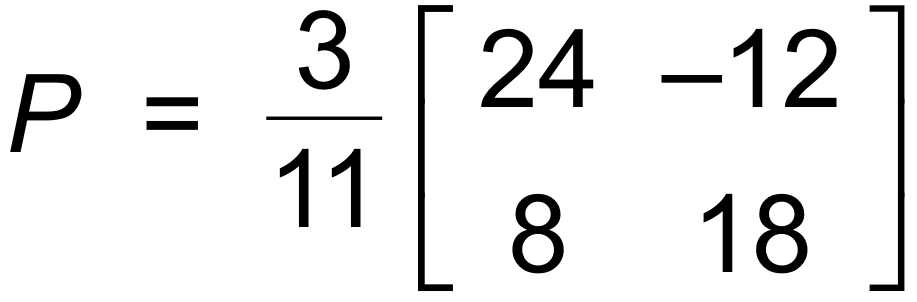
(c) Speed in Easterly direction is  ✓

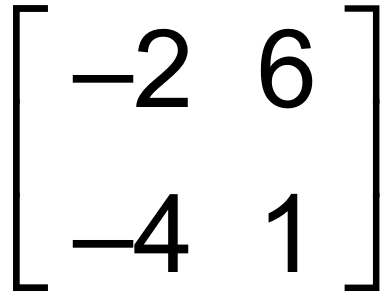
Time taken is  minutes ✓ [8]

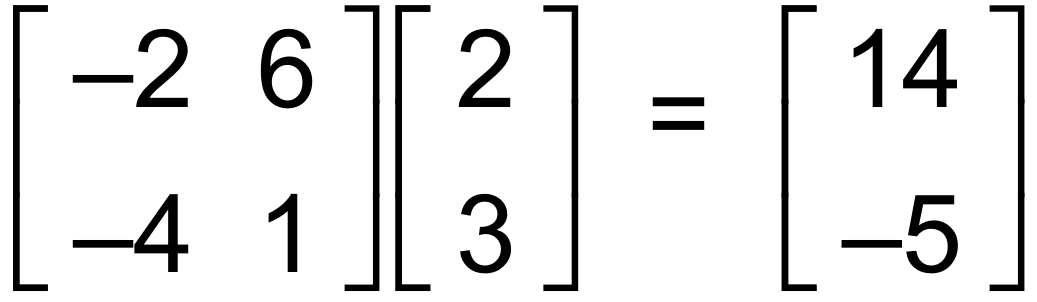
13. (a) (i)  ∴ Rotation of 180° ✓✓

(ii)  ∴ Rotation of 270° clockwise ✓✓

(b) *P* = 6(*B* – 2*A*) x *B* –1 ✓

∴  ✓✓

(c) *BA* = 

*BAX* =  ✓

Co−ordinates are (14, −5) ✓

(d) Det B = 22 ∴ Area = 25 x 22 = 550 ✓✓

(e) Singular matrix has det = 0 ✓

∴ Area = 0 i.e. A line ✓ [13]

14. (a) 3(2**i** + 3**j**) – (*m***i** – 5**j**) = 8**i** + 14**j** ✓

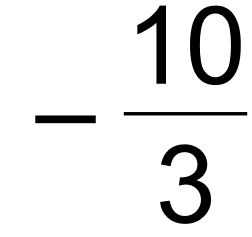
∴ (6 – m)**i** + 14**j** = 8**i** + 14**j**

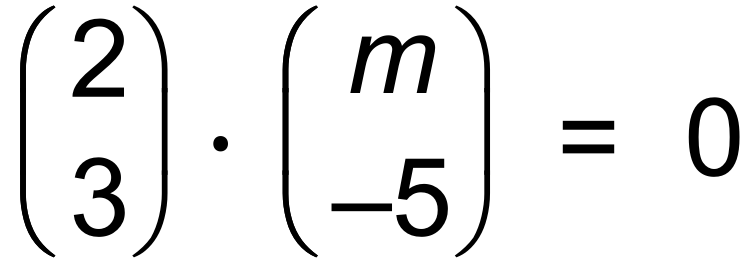
∴ 6 – *m* = 8

∴ *m* = – 2 ✓

(b) 2**i** + 3**j** = *k*(*m***i** – 5**j**) ✓

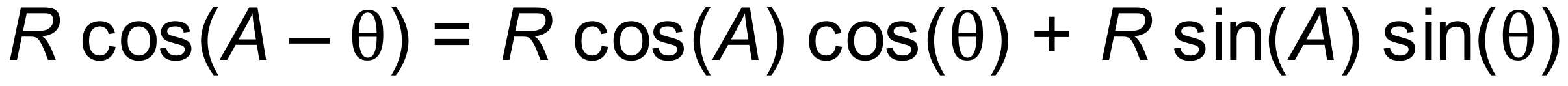
∴ 2 = *km* and 3 = –5*k*

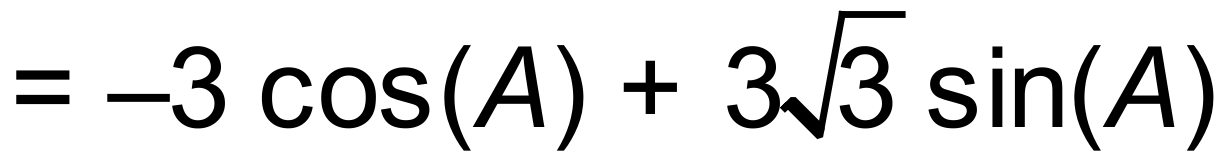
∴ *k* = – 0.6 and by substitution, *m* =  ✓

(c)  ✓

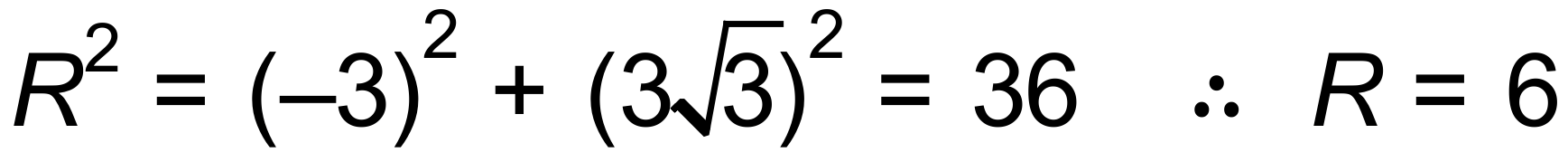
∴ 2*m* – 15 = 0

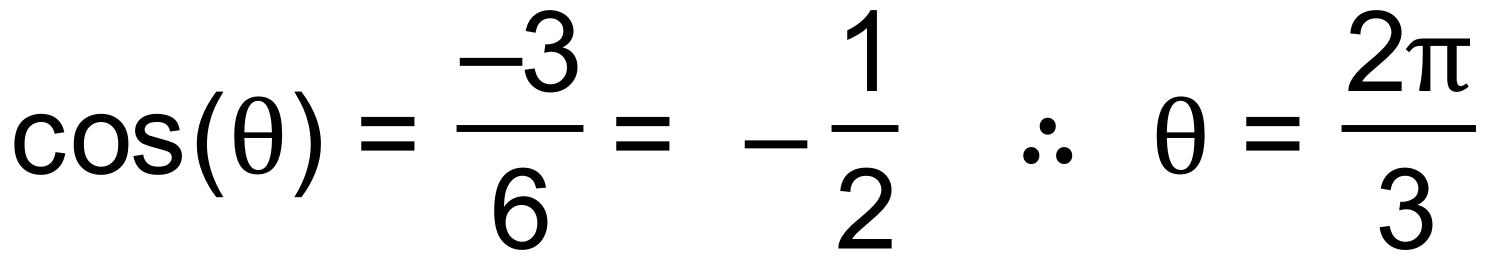
∴ *m* = 7.5 ✓ [6]

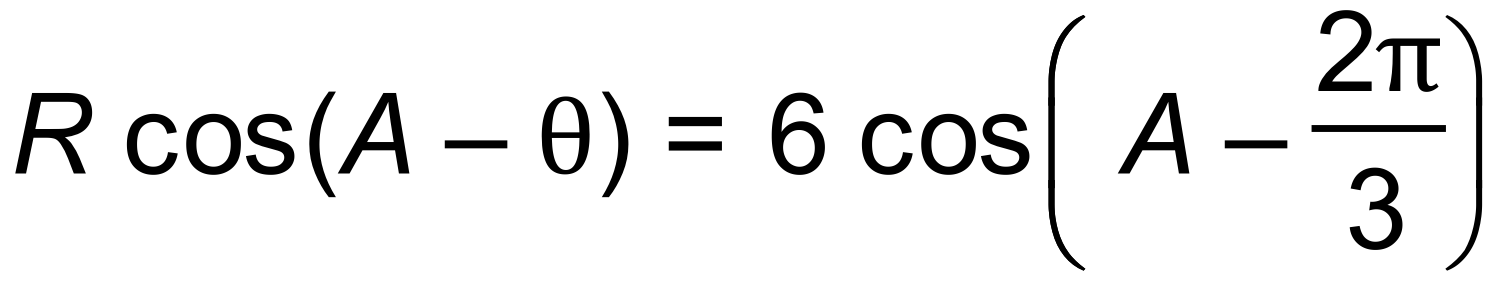
15. (a) 

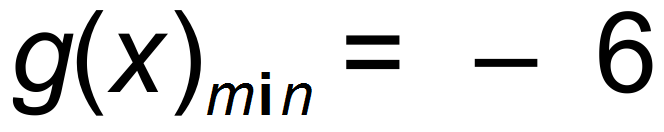


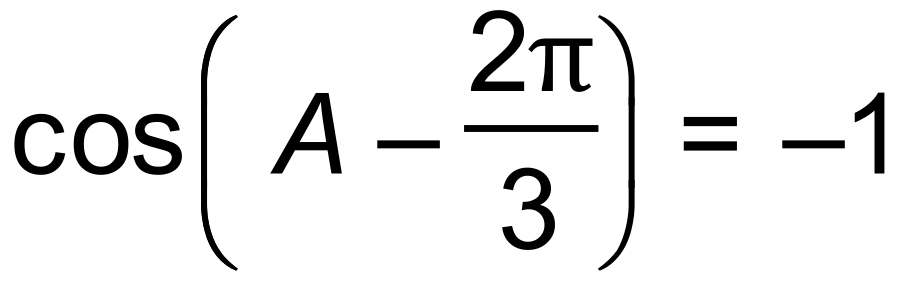


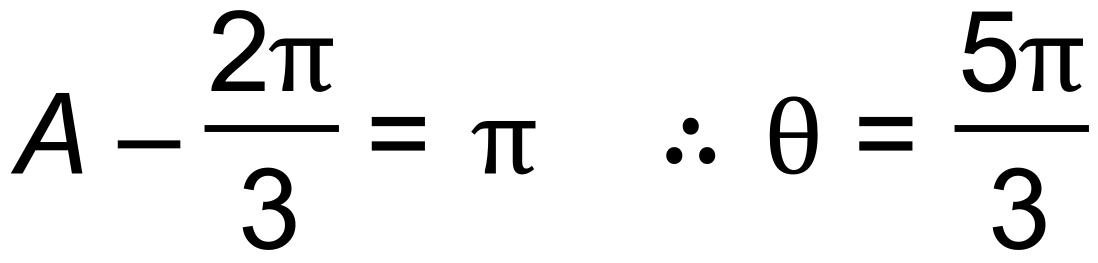
hence,  ✓✓

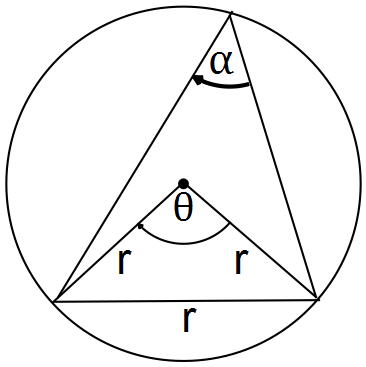
and  ✓

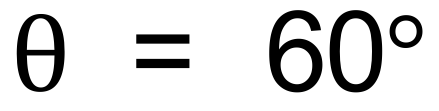
therefore,  ✓

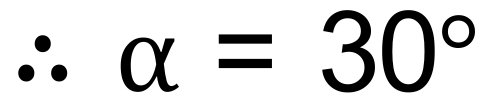
(b) (i)  ✓

(ii) for 

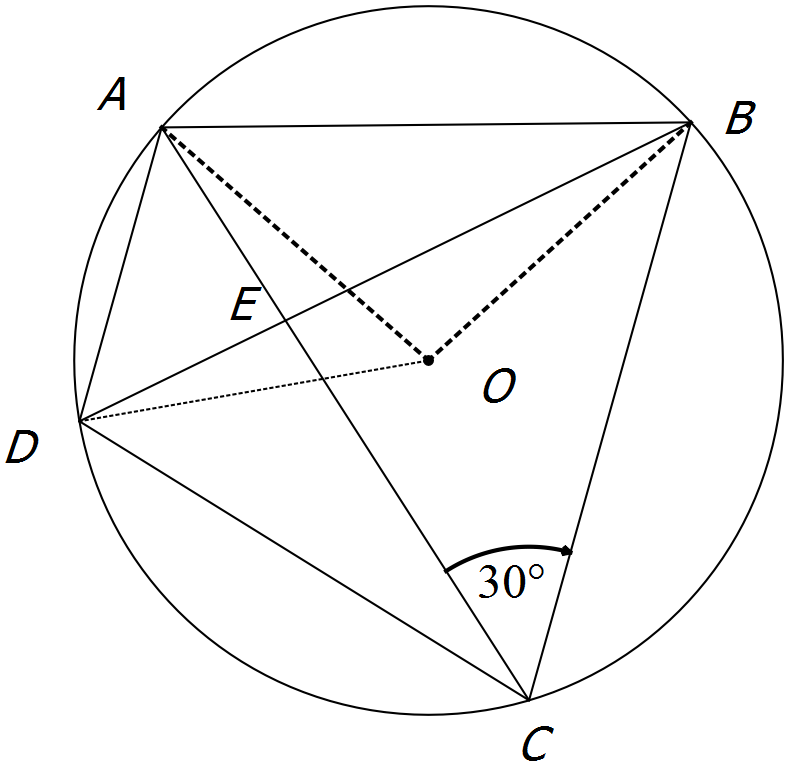
hence  ✓✓ [7]



16. (a)  (equilateral triangle) ✓

 (central angle theorem)✓

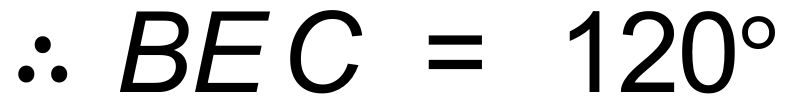
(b) (i)



*AOB* = 60 (proved in (a))

 (theorem)

Similarly, *DBC* = 30



 ✓✓✓

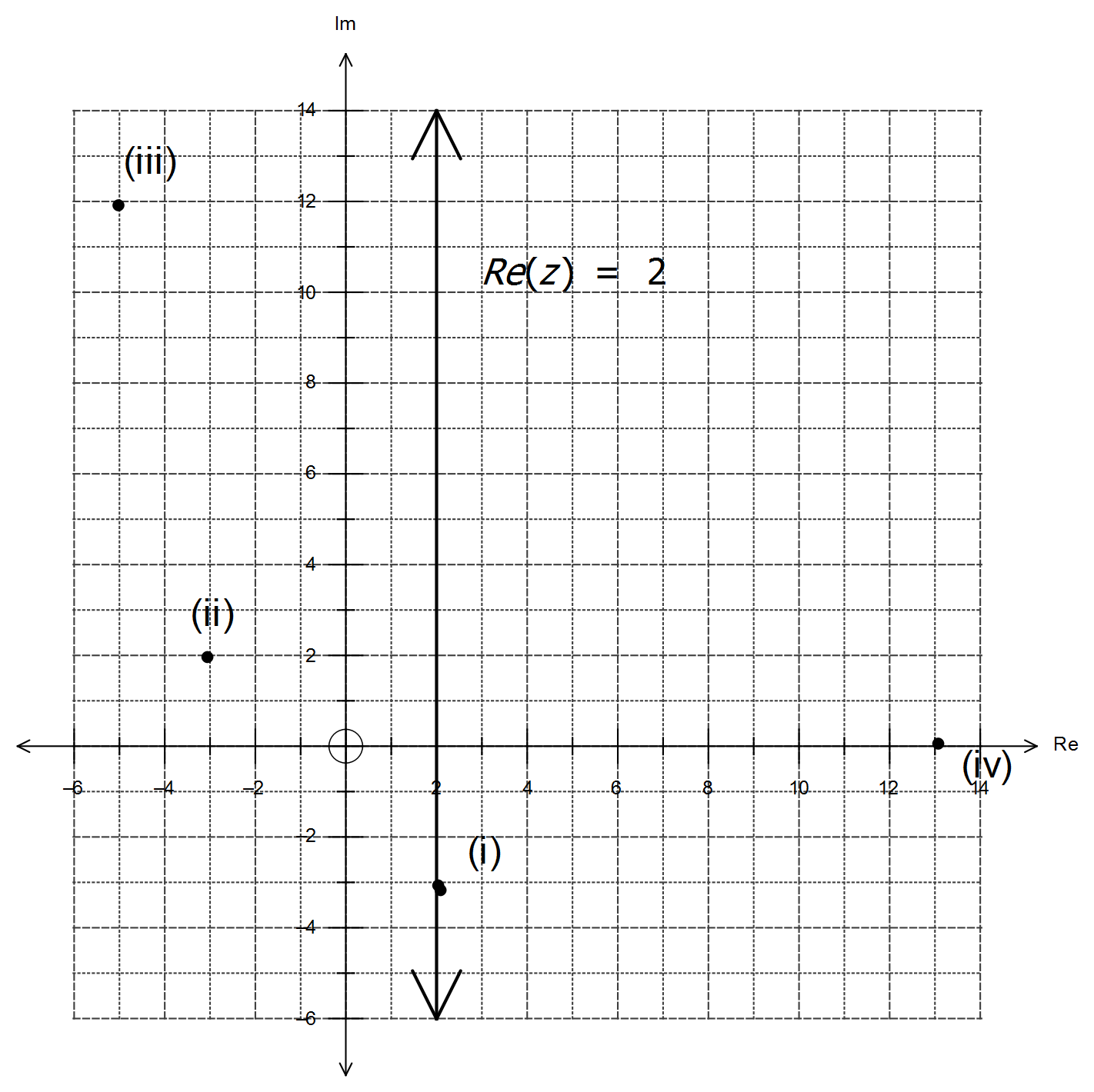
(ii) Assume E is the centre.

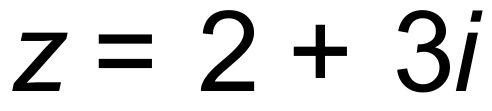
All angles of ABE = 60 

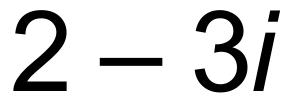
and all angles of BEC = 60 

But AEB = 2ACB which is impossible if they are both 60 

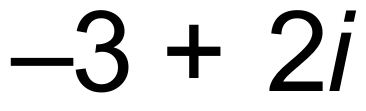
 E is not the centre. ✓✓✓ [8]



17. 

(i)  ✓✓

✓✓

(ii)  ✓✓

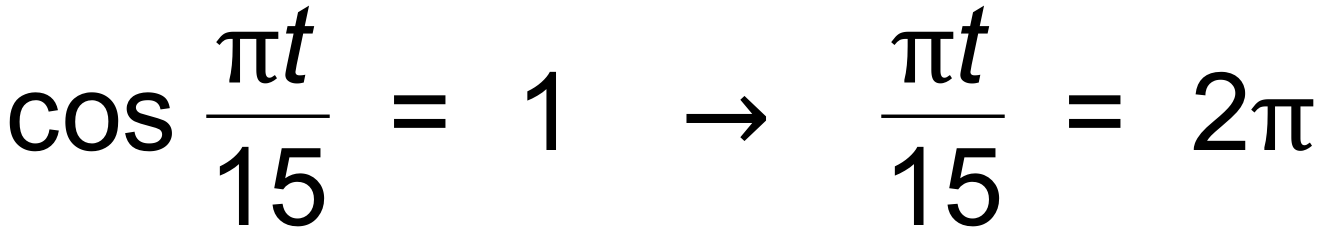
(iii)  ✓✓

(iv) 13✓✓

✓✓

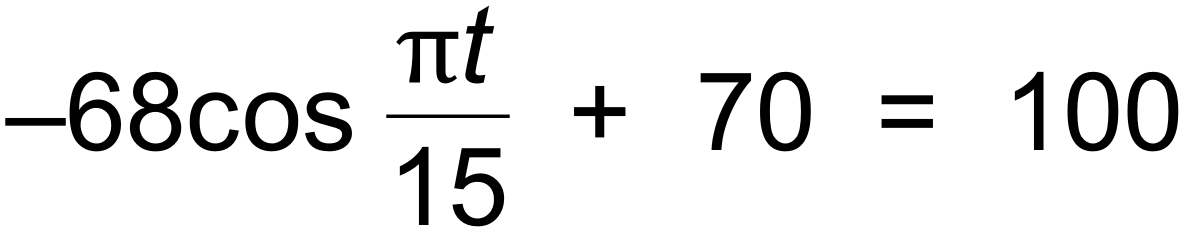
[8]

18. (a) 2m ✓

(b)  ✓

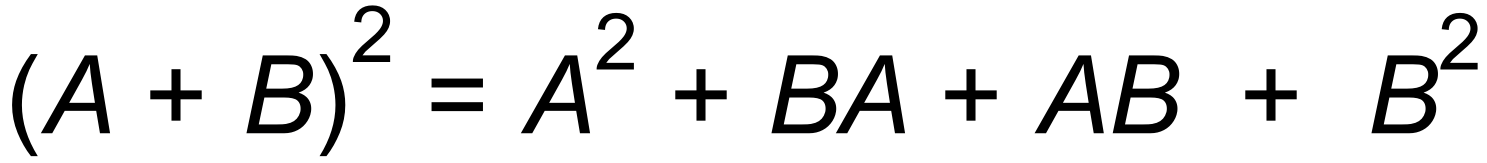
∴ t = 30 secs ✓

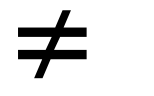
(c) 138m ✓

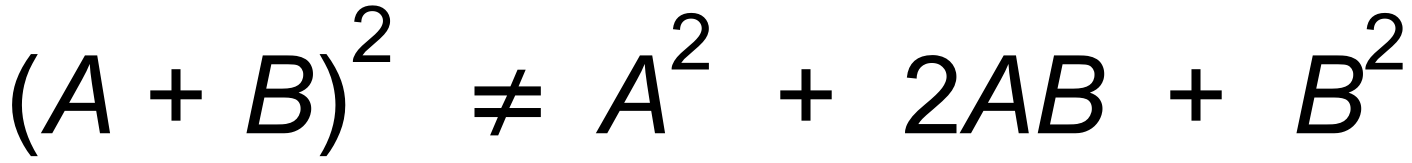
(d)  ✓

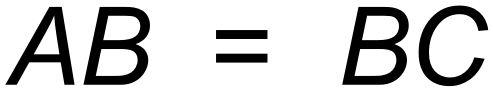
∴ *t* = 9.68, 20.32 ✓

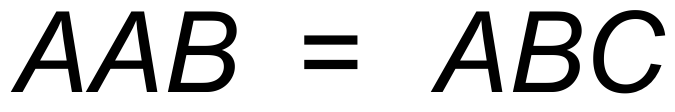
∴ 10.64 minutes ✓ [7]

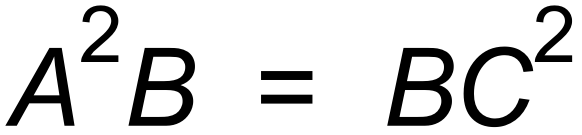
19. (a) It is given that  ✓

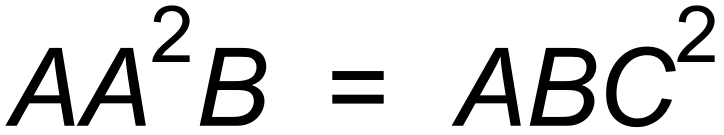
Since *AB*  *BA*, ✓

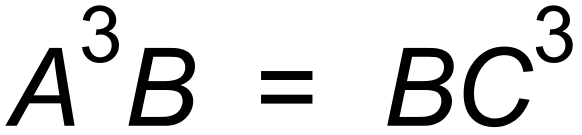
∴ 

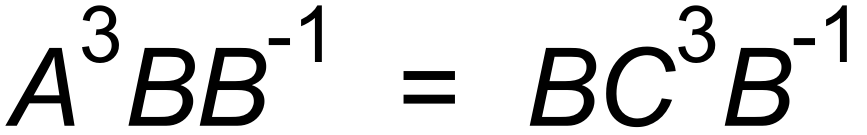
(b) 

∴  ✓

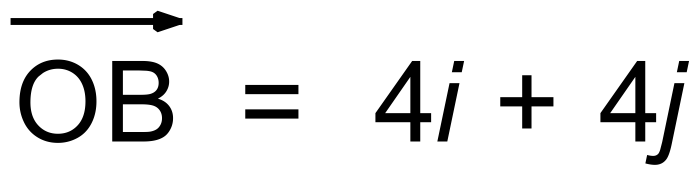
∴ 

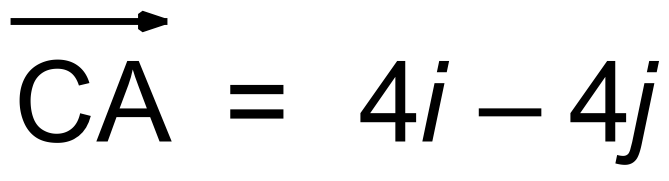
∴  ✓

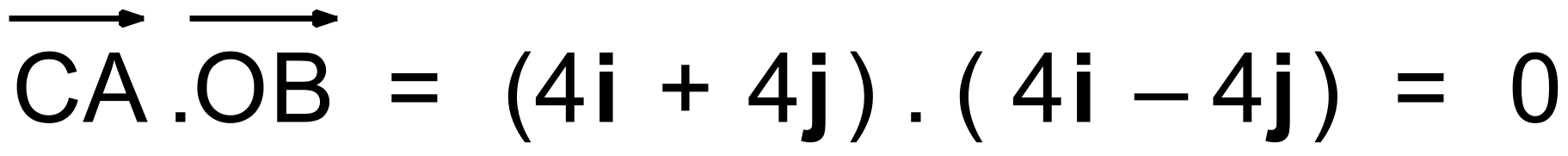
∴ 

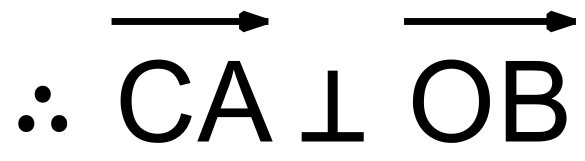
∴  ✓

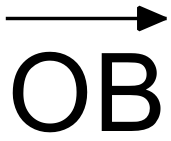
∴  as required ✓ [6]

20. (a)  ✓

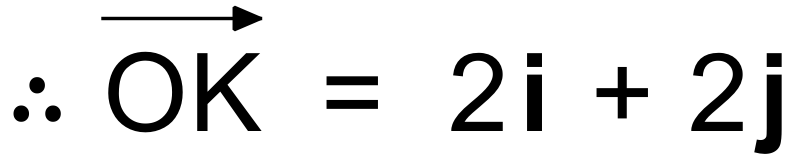
 ✓

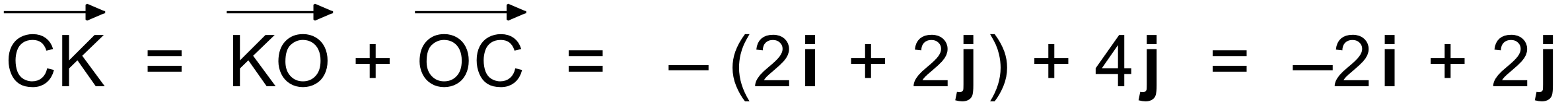
(b)  ✓

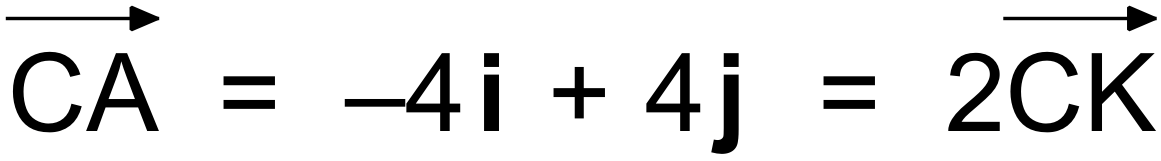
 ✓

(c) Let k be the midpoint of .

Then K = (2, 2)

 ✓

So  ✓

∴ K is the midpoint of  ✓ [7]

∴ Diagonals bisect each other.